Journal of
Mechanical
Science and
Technology

Journal of Mechanical Science and Technology 21 (2007) 745~754

Moving Interfacial Crack Between Two Dissimilar Soft Ferromagnetic Materials in Uniform Magnetic Field

She-Xu Zhao[a], Kang Yong Lee[b*]

[a]*Department of Engineering Mechanics, Shanghai Jiaotong University, Shanghai 200240, China*
[b]*School of Mechanical Engineering, Yonsei University, Seoul 120-749, South Korea*

(Manuscript Received November 20, 2006; Revised March 6, 2007; Accepted March 20, 2007)

Abstract

This paper presents the dynamic magnetoelastic stress intensity factors of a Yoffe-type moving crack at the interface between two dissimilar soft ferromagnetic elastic half-planes. The solids are subjected to a uniform in-plane magnetic field and the crack is opened by internal normal and shear tractions. The problem is considered within the framework of linear magnetoelasticity. By application of the Fourier integral transform, the mixed boundary problem is reduced to a pair of integral equations of the second kind with Cauchy-type singularities. The singular integral equations are solved by means of a Jacobi polynomial expansion method. For a particular case, closed-form solutions are obtained. It is shown that the magnetoelastic stress intensity factors depend on the moving velocity of the crack, the magnetic field and the magnetoelastic properties of the materials.

Keywords: Soft ferromagnetic material; Moving interfacial crack; Dynamic magnetoelastic stress intensity factor

1. Introduction

Over the past years, the magnetoelastic coupling effect in determining the stresses distribution around a crack within a soft ferromagnetic material has been a major concern of many studies, such as, Shindo (1977, 1982), Yeh (1989), Sabir and Maugin (1996), Bagdasarrian and Hasannian (2000), Liang et al. (2000), Lin and Yeh (2002), Zhao and Lee (2004). The same problem in investigating the dynamic behavior of a soft ferromagnetic elastic solid with cracks has also received attention, such as in Shindo (1983, 1984). These works are of practical significance in view of the fact that the materials are increasingly used in engineering devices which are usually subjected to strong magnetic field.

Because of the increasing applications of composite materials in modern technology, it is needed to examine the effect of magnetic field on the stress distribution in bonded dissimilar magnetic materials with crack on the interface. Asanyan et al. (1988) once investigated the interfacial crack problem for a bonded infinite. Recently, Lin et al. (2002) obtained a closed-form solution of local magnetic and magnetoelastic quantities for interfacial cracks between two dissimilar soft ferromagnetic half-planes. Zhao and Lee (2007) solved interfacial crack problem in layered soft ferromagnetic materials in uniform magnetic field. To the authors' knowledge, there is still no attempt devoted to the dynamic problem of an interfacial crack in soft ferromagnetic bodies.

This paper presents a study on the dynamic problem of a Yoffe-type moving crack located at the interface between two dissimilar soft ferromagnetic half-planes under a uniform magnetic field and tractions inside the crack. The study is based on the Pao and Yeh's linear theory of megnetoealsticity.

*Corresponding author. Tel.: +82 2 2123 2813; Fax.: +82 2 2123 2813
E-mail address: KYL2813@yonsei.ac.kr

Application of the Fourier integral transform reduces the problem to two singular integral equations of the second kind for the mixed boundary conditions. Solutions for the magnetoelastic stress intensity factors are obtained and shown graphically for the different moving crack velocity, magnetic field induction and magnetic properties of the materials, respectively.

2. Fundamental equations of the linear magnetoelasticity

Consider the plane strain problem for an isotropic, homogeneous, soft ferromagnetic solid placed in an external magnetic field within the framework of a fixed rectangular coordinate system (X,Y,Z). It is assumed that all field variables of interest are independent of Z. Let B_i, H_i and M_i $(i = X,Y)$ denote the magnetic induction, the magnetic intensity and the magnetization, respectively.

According to Pao and Yeh (1973), the magnetic quantities (B_i, H_i, M_i) are decomposed into two parts associated with those (B_i^0, H_i^0, M_i^0) in a rigid body state and those (b_i, h_i, m_i) in a perturbed state due to deformation, with the assumption that (b_i, h_i, m_i) are of the same order of magnitude as the displacement u_i of the solid.

Without loss in generality, we consider that $B_X^0 = B_0$, $B_Y^0 = 0$, where B_0 = constant being the applied magnetic field. In this case, the magnetoelastic solution for the rigid body state can easily be obtained with $M_i^0 = \chi H_i^0$ and $H_i^0 = B_i^0 / \mu_0 \mu_r$, where $\mu_0 = 4\pi \times 10^{-7}$ (N/A^2) is the absolute permeability of vacuum, χ is the magnetic susceptibility, and $\mu_r = 1 + \chi$ is the relative permeability of the material. Consequently, in what follows we focus our attention on the solution for the perturbed state.

On neglecting the effect of the magnetostriction and assuming that $\left|M_j^0 u_{i,j}\right| \ll \left|m_i\right|$, the equations of motion for the perturbed state can be linearized as (Pao and Yeh , 1973; Shindo, 1983)

$$\nabla^2 u_i + \frac{1}{1-2\nu}\Delta_{,i} + \frac{2\chi b_e^2}{\mu_r} h_{i,k} = \frac{1}{c^2}\frac{\partial^2 u_i}{\partial t^2}, \quad i = X,Y;\ k = Y \tag{1}$$

$$\nabla^2 \varphi = 0, \quad h_i = \frac{B_0}{\mu_0}\varphi_{,i}, \quad i = X,Y \tag{2}$$

in which $\nabla^2 = \partial^2/\partial X^2 + \partial^2/\partial Y^2$ is the two-dimensional Laplacian operator, a comma stands for partial differentiation with respect to the coordinate, $b_e = B_0 \big/ \sqrt{\mu\mu_0}$ is referred to as normalized magnetic induction, $\Delta = u_{X,X} + u_{Y,Y}$, φ is the magnetic potential, ρ, ν and μ are respectively the mass density, the Poisson's ratio and the shear modulus of the material, $c = \sqrt{\mu/\rho}$ and t denote the shear wave speed and the time respectively.

Then the linearized magnetic and magentoelastic constitutive equations in the perturbed state have respectively the forms as

$$b_i = \mu_0(h_i + m_i) = \mu_0\mu_r h_i, \quad m_i = \chi\, h_i \tag{3}$$

$$t_{ij} = \sigma_{ij} + \frac{\chi}{\mu_0\mu_r^2} B_i^0 B_j^0 + \frac{\chi}{\mu_r}\left(B_i^0 h_j + B_j^0 h_i\right) \tag{4}$$

$$t_{ij}^M = \frac{1}{\mu_0\mu_r} B_i^0 B_j^0 - \frac{1}{2\mu_0\mu_r^2} B_k^0 B_k^0 \delta_{ij} + \left(B_i^0 h_j + B_j^0 h_i\right) - \frac{1}{\mu_r} B_k^0 h_k \delta_{ij}$$

$$t_{ij}^T = t_{ij} + t_{ij}^M \tag{5}$$

in which $i,j,k = X,Y$, δ_{ij} is the Kronecker delta, t_{ij}, t_{ij}^M and t_{ij}^T are the magnetoelastic stresses, the Maxwell stresses and the total stresses respectively, and σ_{ij} is the elastic stresses given as

$$\sigma_{ij} = \frac{2\mu\nu}{1-2\nu} u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \tag{6}$$

Note that the implicit summation convention over a repeated index applies for Latin subscripts in this section.

3. Moving interfacial crack problem and the integral equations

As displayed in Fig. 1, consider a Griffith crack along $Y = 0$, $|X| \le a$ and located at the interface between two dissimilar soft ferromagnetic half planes Ω_1 and Ω_2. For convenience, the superscripts "(1)" and "(2)" are used to identify the field quantities in Ω_1 and Ω_2, but the subscripts "1" and "2" to identify the corresponding material properties.

According to Yoffe (1951), it is assumed that the crack moves along the X-axis with uniform velocity V and without change in length, and that a coordinate system (x,y,z) attached to the crack center moves simultaneously with the same velocity. Therefore, it is convenient to study the crack propagation in the moving coordinate system for a better comprehension to its essential dynamic features.

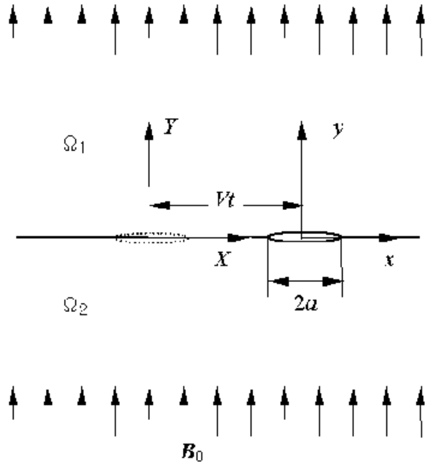

Fig. 1. Two bonded dissimilar soft ferromagnetic half-planes with a moving interfacial crack in a uniform magnetic field.

By applying the Galelian transformation $x = X - Vt$, $y = Y$ to Eqs. (1) and (2), the field equations for the present solids have the forms as

$$\left(\frac{2(1-\nu_j)}{(1-2\nu_j)}-\frac{V^2}{c_j^2}\right)\frac{\partial^2 u_x^{(j)}}{\partial x^2}+\frac{\partial^2 u_x^{(j)}}{\partial y^2}+\frac{1}{(1-2\nu_j)}\frac{\partial^2 u_y^{(j)}}{\partial x\partial y}+\frac{2\chi_j b_{ej}^2}{\mu_{rj}}\varphi_{,xy}^{(j)}=0$$

$$\frac{2(1-\nu_j)}{(1-2\nu_j)}\frac{\partial^2 u_y^{(j)}}{\partial y^2}+\left(1-\frac{V^2}{c_j^2}\right)\frac{\partial^2 u_y^{(j)}}{\partial x^2}+\frac{1}{(1-2\nu_j)}\frac{\partial^2 u_x^{(j)}}{\partial x\partial y}+\frac{2\chi_j b_{ej}^2}{\mu_{rj}}\varphi_{,yy}^{(j)}=0 \tag{7}$$

$$\nabla^2\varphi^{(j)}=0,\quad h_x^{(j)}=\frac{B_0}{\mu_0}\varphi_{,x}^{(j)},\quad h_y^{(j)}=\frac{B_0}{\mu_0}\varphi_{,y}^{(j)} \tag{8}$$

in which $u_x^{(j)}(x,y)=u_X^{(j)}(X,Y,t\}$, $u_y^{(j)}(x,y)=u_Y^{(j)}(X,Y,t\}$, $\varphi^{(j)}(x,y)=\varphi^{(j)}(X,Y,t\}$, $h_x^{(j)}(x,y)=h_X^{(j)}(X,Y,t\}$, $h_y^{(j)}(x,y)=h_Y^{(j)}(X,Y,t\}$ and $j=1,2$.

In addition to action of the uniform magnetic field B_0, let the surfaces of the crack be subjected to known normal pressure $p_1(x)$ and shear traction $p_2(x)$. Owing to the current problem being symmetric with respect to the $x=0$ plane, it is sufficient to treat it for $x \ge 0$ only. Upon assuming that two solids are bonded perfectly, the continuity conditions on the stresses and the displacements are given as (Pao and Yeh, 1973)

$$t_{yx}^{(1)}(x,\ 0)=t_{yx}^{(2)}(x,\ 0),\quad 0\le x<\infty \tag{9}$$

$$t_{yy}^{(1)}(x,0)-t_{yy}^{(2)}(x,0)=\frac{1}{2}\left(\frac{\mu_1\chi_1^2 b_{e1}^2}{\mu_{r1}^2}-\frac{\mu_2\chi_2^2 b_{e2}^2}{\mu_{r2}^2}\right)+\left(\frac{\mu_1\chi_1^2 b_{e1}^2}{\mu_{r1}}\varphi_{,y}^{(1)}(x,0)-\frac{\mu_2\chi_2^2 b_{e2}^2}{\mu_{r2}}\varphi_{,y}^{(2)}(x,0)\right),\quad 0\le x<\infty \tag{10}$$

$$\mu_{r1}\varphi_{,y}^{(1)}(x,0)=\mu_{r2}\varphi_{,y}^{(2)}(x,0),\quad 0\le x<\infty \tag{11}$$

$$\varphi_{,x}^{(1)}(x,0)-\varphi_{,x}^{(2)}(x,0)=\frac{\chi_1}{\mu_{r1}}u_{y,x}^{(1)}(x,0)-\frac{\chi_2}{\mu_{r2}}u_{y,x}^{(2)}(x,0),\quad 0\le x<\infty \tag{12}$$

And the mixed boundary conditions are

$$t_{yy}^{(1)}(x,0)=\frac{1}{2}\frac{\mu_1\chi_1^2 b_{e1}^2}{\mu_{r1}^2}+\frac{\mu_1\chi_1^2 b_{e1}^2}{\mu_{r1}}\varphi_{,y}^{(1)}(x,0)-p_1(x),\quad 0\le x<a$$

$$u_y^{(1)}(x,0)=u_y^{(2)}(x,0),\quad a\le x<\infty \tag{13}$$

$$t_{yx}^{(1)}(x,0)=-p_2(x),\quad 0\le x<a$$

$$u_x^{(1)}(x,0)=u_x^{(2)}(x,0),\quad a\le x<\infty \tag{14}$$

Now, the Fourier integral transform with respect to x and the inverse transform are applied to Eqs. (7) and (8), giving rise to

$$u_x^{(1)}(x,y)=\frac{2}{\pi}\int_0^\infty\frac{1}{\tau}\left[A_1^{(1)}(\tau)\,\mathrm{e}^{-\gamma_1^{(1)}\tau y}+A_2^{(1)}(\tau)\,\mathrm{e}^{-\gamma_2^{(1)}\tau y}\right]\sin\tau x\,\mathrm{d}\tau \tag{15}$$

$$u_y^{(1)}(x,y)=\frac{2}{\pi}\int_0^\infty\frac{1}{\tau}\left[\theta_1^{(1)}A_1^{(1)}(\tau)\,\mathrm{e}^{-\gamma_1^{(1)}\tau y}+\theta_2^{(1)}A_2^{(1)}(\tau)\,\mathrm{e}^{-\gamma_2^{(1)}\tau y}-\frac{2\chi_1 b_{e1}^2(1-2\nu_1)}{\mu_{r1}}A_3^{(1)}(\tau)\,\mathrm{e}^{-\gamma_2^{(1)}\tau y}\right]\cos\tau x\,\mathrm{d}\tau \tag{16}$$

$$\varphi^{(1)}(x,y)=\frac{2}{\pi}\int_0^\infty\frac{1}{\tau}A_3^{(1)}(\tau)\mathrm{e}^{-\gamma_2^{(1)}\tau y}\cos\tau x\,\mathrm{d}\tau-g^{(1)}y \tag{17}$$

and

$$u_x^{(2)}(x,y)=\frac{2}{\pi}\int_0^\infty\frac{1}{\tau}\left[A_1^{(2)}(\tau)\,\mathrm{e}^{\gamma_1^{(2)}\tau y}\right.$$

$$+ A_2^{(2)}(\tau)\, e^{\gamma_2^{(2)}\tau y} \Big] \sin \tau x \, d\tau \tag{18}$$

$$u_y^{(2)}(x, y) = \frac{2}{\pi} \int_0^\infty \frac{1}{\tau} \Big[\theta_1^{(2)} A_1^{(2)}(\tau)\, e^{\gamma_1^{(2)}\tau y} + \theta_2^{(2)} A_2^{(2)}(\tau)\, e^{\gamma_2^{(2)}\tau y} \tag{19}$$

$$- \frac{2\chi_2 b_{e2}^2 (1-2\nu_2)}{\mu_{r2}} A_3^{(2)}(\tau)\, e^{\gamma_2^{(2)}\tau y} \Big] \cos \tau x \, d\tau$$

$$\varphi^{(2)}(x, y) = \frac{2}{\pi} \int_0^\infty \frac{1}{\tau} A_3^{(2)}(\tau) e^{\gamma_2^{(2)}\tau y} \cos \tau x \, d\tau - g^{(2)} y \tag{20}$$

in which $A_j^{(1)}$ and $A_j^{(2)}$ $(j = 1, 2, 3)$ are unknown functions and $g^{(n)}$ $(n = 1, 2)$ are real constants, which are to be determined by the continuity conditions, and

$$\gamma_1^{(j)} = \sqrt{\frac{1}{2(1-\nu_j)} \left[1 + (1-2\nu_j)(1 - \frac{V^2}{c_j^2}) \right]},$$

$$\gamma_2^{(j)} = \sqrt{(1 - \frac{V^2}{c_j^2})} \tag{21}$$

$$\theta_n^{(j)} = (-1)^j \left\{ \frac{1}{\gamma_n^{(j)}} \left[1 + (1-2\nu_j)\left(1 - \frac{V^2}{c_j^2}\right) \right] - (1-2\nu_j)\gamma_n^{(j)} \right\}$$

where $n, j = 1, 2$.

With the aid of the Galelian transformation and Eqs. (15)-(20), the magnetoelastic stresses, given by Eq. (4) and (6), in the bonded solids are then reduced to

$$t_{yy}^{(1)}(x, y) = \frac{2}{\pi} \int_0^\infty 2\mu_1 \Big[k_1 A_1^{(1)}(\tau) e^{-\gamma_1^{(1)}\tau y} + \left(k_2 A_2^{(1)}(\tau) + k_3 A_3^{(1)}(\tau) \right) e^{-\gamma_2^{(1)}\tau y} \Big] \cos \tau x \, d\tau \tag{22}$$

$$- \frac{2\mu_1 \chi_1 b_{e1}^2}{\mu_{r1}} g^{(1)} + \frac{\mu_1 \chi_1 b_{e1}^2}{\mu_{r1}^2}$$

$$t_{yx}^{(1)}(x, y) = \frac{2}{\pi} \int_0^\infty - \mu_1 \Big[k_4 A_1^{(1)}(\tau) e^{-\gamma_1^{(1)}\tau y} + \left(k_5 A_2^{(1)}(\tau) + k_6 A_3^{(1)}(\tau) \right) e^{-\gamma_2^{(1)}\tau y} \Big] \sin \tau x \, d\tau \tag{23}$$

and

$$t_{yy}^{(2)}(x, y) = \frac{2}{\pi} \int_0^\infty 2\mu_2 \Big[k_7 A_1^{(2)}(\tau)\, e^{\gamma_1^{(2)}\tau y} + \left(k_8 A_2^{(2)}(\tau) + k_9 A_3^{(2)}(\tau) \right) e^{\gamma_2^{(2)}\tau y} \Big] \cos \tau x \, d\tau \tag{24}$$

$$- \frac{2\mu_2 \chi_2 b_{e2}^2}{\mu_{r2}} g^{(2)} + \frac{\mu_2 \chi_2 b_{e2}^2}{\mu_{r2}^2}$$

$$t_{yx}^{(2)}(x, y) = \frac{2}{\pi} \int_0^\infty \mu_2 \Big[k_{10} A_1^{(2)}(\tau)\, e^{\gamma_1^{(2)}\tau y} + \left(k_{11} A_2^{(2)}(\tau) + k_{12} A_3^{(2)}(\tau) \right) e^{\gamma_2^{(2)}\tau y} \Big] \sin \tau x \, d\tau \tag{25}$$

where the constants k_n $(n = 1, 2, \cdots 12)$ depend on the velocity of the moving crack and the properties of the materials and are given in the Appendix.

In order to determine the unknown functions and reduce the mixed boundary conditions into integral equations, we first define the following dislocation densities

$$\frac{\partial}{\partial x} \left[u_x^{(1)}(x, 0) - u_x^{(2)}(x, 0) \right] = \psi_1(x) \tag{26}$$

$$\frac{\partial}{\partial x} \left[u_y^{(1)}(x, 0) - u_y^{(2)}(x, 0) \right] = \psi_2(x) \tag{27}$$

Then the displacement continuity conditions outside the crack and the displacement single-valuedness conditions can be written as

$$\psi_j(x) = 0, \quad a < |x| < \infty, \quad \int_{-a}^{a} \psi_j(s) ds = 0, \quad j = 1, 2 \tag{28}$$

Next, inserting Eqs. (15)~(20) and (22)~(25) into Eqs. (9)~(12), $(13)_2$~$(14)_2$ and (26)~(27), after some lengthy manipulations, it turns out that

$$A_n^{(j)}(\tau) = f_{n1}^{(j)} \int_0^a \psi_1(s) \cos \tau s ds + f_{n2}^{(j)} \int_0^a \psi_2(s) \sin \tau s ds, \quad n = 1, 2, 3;\ j = 1, 2 \tag{29}$$

$$g^{(1)} = \frac{1}{2\mu_{r1}}, \quad g^{(2)} = \frac{1}{2\mu_{r2}} \tag{30}$$

where $f_{n1}^{(j)}$ and $f_{n2}^{(j)}$ are known constants and are given in the Appendix. It can now be seen that from Eqs. (29)~(30), the stresses given by Eqs. (22)~(25) may be expressed in terms of the density functions $\psi_1(s)$ and $\psi_2(s)$.

Finally, Eqs. $(13)_1$ and $(14)_1$, together with Eqs. (15)~(16), (18)~(19) and (29)~(30), yield the following integral equations

$$\frac{2}{\pi}\int_0^\infty E_{11}\cos\tau x\int_0^a \psi_1(s)\cos\tau s\,\mathrm{d}s\,\mathrm{d}\tau + \frac{2}{\pi}\int_0^\infty E_{12}\cos\tau x\int_0^a \psi_2(s)\sin\tau s\,\mathrm{d}s\,\mathrm{d}\tau = -\frac{p_1(x)}{2\mu_1}, \quad |x|<a \tag{31}$$

$$\frac{2}{\pi}\int_0^\infty E_{21}\cos\tau x\int_0^a \psi_1(s)\cos\tau s\,\mathrm{d}s\,\mathrm{d}\tau + \frac{2}{\pi}\int_0^\infty E_{22}\cos\tau x\int_0^a \psi_2(s)\sin\tau s\,\mathrm{d}s\,\mathrm{d}\tau = \frac{p_2(x)}{\mu_1}, \quad |x|<a \tag{32}$$

in which

$$E_{1n} = k_1 f_{1n}^{(1)} + k_2 f_{2n}^{(1)} + \bar{k}_3 f_{3n}^{(1)}$$
$$E_{2n} = k_4 f_{1n}^{(1)} + k_5 f_{2n}^{(1)} + k_6 f_{3n}^{(1)}, \quad n=1,2 \tag{33}$$

where $\bar{k}_3$ is a known constant given in the Appendix.

By interchanging the integration order in Eqs.(31) and (32), it allows us to express the singular integral equations in the dislocation densities as

$$\lambda_1\psi_1(x) + \frac{1}{\pi}\int_{-a}^a \frac{\psi_2(s)}{s-x}\mathrm{d}s = -\frac{p_1(x)}{2\mu_1 E_{12}}, \quad |x|<a \tag{34}$$

$$\lambda_2\psi_2(x) - \frac{1}{\pi}\int_{-a}^a \frac{\psi_1(s)}{s-x}\mathrm{d}s = \frac{p_2(x)}{\mu_1 E_{21}}, \quad |x|<a \tag{35}$$

in which

$$\lambda_1 = \frac{E_{11}}{E_{12}}, \qquad \lambda_2 = \frac{E_{22}}{E_{21}} \tag{36}$$

In Eqs. (34) and (35), only the simple Cauchy-type kernel $1/(s-x)$ contributes to the singular behavior of the solutions to the integral equations (Muskhelishvili, 1953). It should be noted that Eqs. (34) and (35) must be solved under the displacement single-valuedness conditions, Eq. (28).

4. Solution of the integral equations

Begin by introducing

$$\tilde{\psi}_k(\eta) = \sqrt{\hat{\lambda}_1}\psi_1(\eta) - ir_k\sqrt{\hat{\lambda}_2}\psi_2(\eta)$$
$$\tilde{p}_k(\zeta) = \frac{1}{2\mu_1}\left[\frac{1}{\sqrt{\hat{E}_{11}E_{12}}}p_1(\zeta) + \frac{2ir_k}{\sqrt{\hat{E}_{22}E_{21}}}p_2(\zeta)\right],$$
$$k=1,2 \tag{37}$$

in which $\zeta = x/a$, $\eta = s/a$ are the normalized interval, $r_1 = 1$, $r_2 = -1$ and $i=\sqrt{-1}$. Note that for the current problem, $E_{11}<0$, $E_{22}<0$, $E_{12}>0$ and $E_{21}>0$. In Eq. (37) $\hat{\lambda}_1 = -\lambda_1$, $\hat{\lambda}_2 = -\lambda_2$, $\hat{E}_{11} = -E_{11}$ and $\hat{E}_{22} = -E_{22}$.

Then, the two integral Eqs. (34) and (35) can be combined as

$$\tilde{\psi}_k(\zeta) + \frac{1}{r_k\sqrt{\lambda_1\lambda_2}}\frac{1}{\pi i}\int_{-1}^1 \frac{\tilde{\psi}_k(\eta)}{\eta-\zeta}\mathrm{d}\eta = \tilde{p}_k(\zeta),$$
$$k=1,2, \quad |\zeta|<1 \tag{38}$$

Equations in (38) are the singular integral equations of the second kind. Their solutions have integrable singularities at the end points $\zeta = \pm 1$ and have been extensively investigated by Muskhelishvili (1953). Here, a method in Erdogan (1984) will be used.

As in Erdogan (1984), solutions to the equations in (38) may be approximated by series of the Jacobi polynomials as

$$\tilde{\psi}_k(\zeta) = \frac{1}{2\mu_1}\sum_{n=0}^\infty C_{kn}w_k(\zeta)P_n^{(\alpha_k,\beta_k)}(\zeta),$$
$$|\zeta|<1, \quad k=1,2 \tag{39}$$

where C_{kn} are unknown constants to be determined, $P_n^{(\alpha_k,\beta_k)}(\zeta)$ are the nth order Jacobi polynomials, and $w_k(\zeta)$ are the corresponding weight functions which characterize the singular behavior of $\tilde{\psi}_k(\zeta)$ at $\zeta=\pm 1$ and are given as (Muskhelishvili, 1953)

$$w_k(\zeta) = (1-\zeta)^{\alpha_k}(1+\zeta)^{\beta_k} \tag{40}$$

with

$$\alpha_k = -\frac{1}{2} + ir_k\omega, \quad \beta_k = -\frac{1}{2} - ir_k\omega,$$

$$\omega = \frac{1}{2\pi}\ln\left|\frac{1+\sqrt{\lambda_1\lambda_2}}{1-\sqrt{\lambda_1\lambda_2}}\right|, \quad k=1,2 \tag{41}$$

On account of the orthogonality of Jacobi polynomials (Gradshteyn and Ryzhik, 1980),

$$\int_{-1}^{1} w(\zeta)P_n^{(\alpha,\beta)}(\zeta)P_m^{(\alpha,\beta)}(\zeta)\,\mathrm{d}\zeta = \begin{cases} 0, & n \neq m;\ n,m = 0,1,2,\cdots \\ \theta_m^{(\alpha,\beta)}, & n = m \end{cases} \tag{42}$$

$$\theta_n^{(\alpha,\beta)} = \frac{2^{(\alpha+\beta+1)}}{2n+\alpha+\beta+1}\frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{n!\Gamma(n+\alpha+\beta+1)}$$

together with $P_0^{(\alpha,\beta)}(\zeta)=1$, it can be concluded that the single-valuedness conditions in (28) are identically satisfied provided that $C_{10}=C_{20}=0$.

Substituting Eq. (39) into Eq. (38) and employing the following integral formulas (Karpenko, 1966),

$$\frac{1}{\pi i}\int_{-1}^{1}\frac{w_k(\eta)P_n^{(\alpha_k,\beta_k)}(\eta)}{(\eta-\zeta)}\mathrm{d}\eta = \begin{cases} -r_k\sqrt{\lambda_1\lambda_2}w_k(\zeta)P_n^{(\alpha_k,\beta_k)}(\zeta) + \dfrac{\sqrt{1-\lambda_1\lambda_2}}{2i}P_{n-1}^{(-\alpha_k,-\beta_k)}(\zeta), & |\zeta|<1 \\ -(1-r_k\sqrt{\lambda_1\lambda_2})[-w_k(\zeta)P_n^{(\alpha_k,\beta_k)}(\zeta)+G_{kn}^{\infty}(\zeta)], & |\zeta|>1 \end{cases} \tag{43}$$

where $G_{kn}^{\infty}(\zeta)$ is the principal part of $w_k(\zeta)P_n^{(\alpha_k,\beta_k)}(\zeta)$ at infinity, the singularity in Eq. (38) is removed such that

$$\sum_{n=1}^{\infty}\frac{\sqrt{1-\lambda_1\lambda_2}}{2r_k i\sqrt{\lambda_1\lambda_2}}P_{n-1}^{(-\alpha_k,-\beta_k)}(\zeta)C_{kn} = q_k(\zeta), \quad k=1,2;\ |\zeta|<1 \tag{44}$$

where

$$q_k(\zeta) = 2\mu_1\widetilde{p}_k(\zeta) \tag{45}$$

With the help of following weight functions,

$$(1-\zeta)^{-\alpha_k}(1+\zeta)^{-\beta_k}P_n^{(-\alpha_k,-\beta_k)}(\zeta),\ n=0,1,2,\cdots \tag{46}$$

Eq. (44) can be reduced to an infinite system of algebraic equations in the unknown constants C_{kn} by using a weighted residual technique (Erdogan and Gupta, 1971). Once the constants C_{kn} are determined, the approximate solutions to $\psi_1(x)$ and $\psi_2(x)$, and then to other field quantities, are readily obtained.

Now, the special case of $p_1(x)=\sigma_0$, $p_2(x)=0$, where σ_0 is constant, is considered. Multiplying both sides of Eq. (44) by the Eq. (46) and integrating in the interval $(-1, 1)$, it is found that

$$C_{k1} = ir_k\frac{2\sigma_0\sqrt{\lambda_1\lambda_2}}{\sqrt{\hat{E}_{11}E_{12}(1-\lambda_1\lambda_2)}}, \quad C_{kn}=0, \quad n=2,3,\cdots;\ k=1,2 \tag{47}$$

which by Eq. (39) yields the closed-form solutions as

$$\widetilde{\psi}_k = \frac{ir_k}{\mu_1}\frac{\sigma_0\sqrt{\lambda_1\lambda_2}}{\sqrt{\hat{E}_{11}E_{12}(1-\lambda_1\lambda_2)}}w_k(\zeta)P_1^{(\alpha_k,\beta_k)}(\zeta), \quad k=1,2 \tag{48}$$

From Eqs. (5), (22)~(25), together with Eqs. (29)~(30), (48) and the formulas (43), one can obtain the total magnetoelastic stresses near the crack tip. Then, the magnetoelastic stress intensity factors K_I and K_{II} may be obtained as (Erdogan and Gupta, 1971)

$$\begin{aligned}\frac{\sqrt{\hat{\lambda}_2}}{\widetilde{E}_{12}}\frac{K_I}{\sqrt{a}} + ir_k\frac{2\sqrt{\hat{\lambda}_1}}{\widetilde{E}_{21}}\frac{K_{II}}{\sqrt{a}} &= \lim_{\zeta\to1^+}(\zeta-1)^{-\alpha_k}(\zeta+1)^{-\beta_k}\left[\frac{\sqrt{\hat{\lambda}_2}}{\widetilde{E}_{12}}t_{yy}^{T(1)}(\zeta,0) + ir_k\frac{2\sqrt{\hat{\lambda}_1}}{\widetilde{E}_{21}}t_{yx}^{T(1)}(\zeta,0)\right] \\ &= \sigma_0\sqrt{\frac{\hat{\lambda}_1\hat{\lambda}_2}{\hat{E}_{11}E_{12}}}(1-2ir_k\omega), \quad k=1,2\end{aligned} \tag{49}$$

where

$$\widetilde{E}_{12} = E_{12} - \frac{\mu_{r1}b_{e1}^2\gamma_2^{(1)}}{2}f_{32}^{(1)}, \quad \widetilde{E}_{21} = E_{21} + b_{e1}^2 f_{31}^{(1)} \tag{50}$$

5. Results and discussions

In what follows, the mechanical properties of the materials are prescribed as $\nu_1 = 0.25$, $\nu_2 = 0.30$,

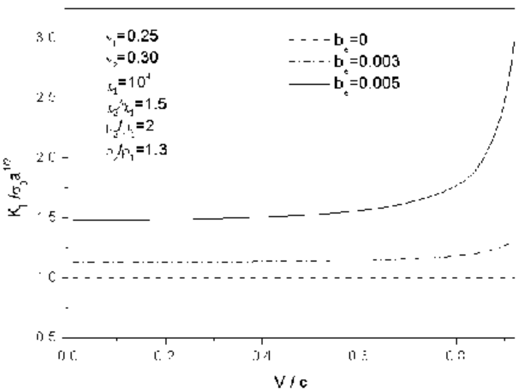

Fig. 2. Stress intensity factor $K_I / \sqrt{a}\sigma_0$ versus V/c for different values of b_e.

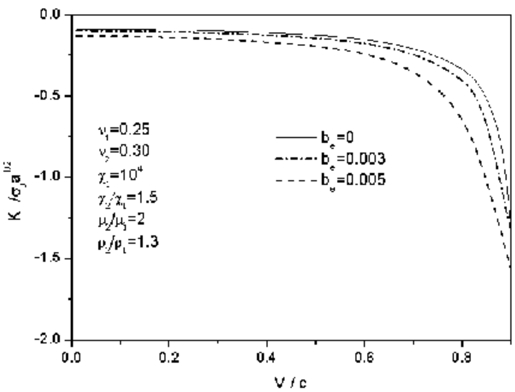

Fig. 3. Stress intensity factor $K_{II} / \sqrt{a}\sigma_0$ versus V/c for different values of b_e.

$\mu_2 / \mu_1 = 2$ and $\rho_2 / \rho_1 = 1.3$. On assuming that the magnetic susceptibilities corresponding to the upper half-plane and the lower one take various values and different combinations, the variations of the magnetoelastic stress intensity factors versus the velocity of the moving crack or the normalized magnetic induction are investigated. Let $V/c = V/c_1 = \sqrt{\rho_1\mu_2/\rho_2\mu_1}(V/c_2)$, $b_e = b_{e2} = b_{e1}\sqrt{\mu_1/\mu_2}$.

First, for $\chi_1 = 10^4$ and $\chi_2 = 1.5 \times 10^4$, the variations of $K_I / \sqrt{a}\sigma_0$ and $K_{II} / \sqrt{a}\sigma_0$ with the changing of V/c at $b_e = 0$, 0.003, 0.005 are examined. The results are displayed in Figs. 2 and 3. Under a prescribed magnetic induction, it is seen that $K_I / \sqrt{a}\sigma_0$ increases while $K_{II} / \sqrt{a}\sigma_0$ decreases with increasing V/c. The stronger the magnetic field, the larger the magnitude of the stress intensity factors and their changing rate are. Especially at larger V/c, the effects due to the magnetic field appear more pronounced. At $b_e = 0$, $K_I / \sqrt{a}\sigma_0$ is independent of V/c, and as $V/c \to 0$ in this case the results coincide with those in Erdogan and Gupta (1971).

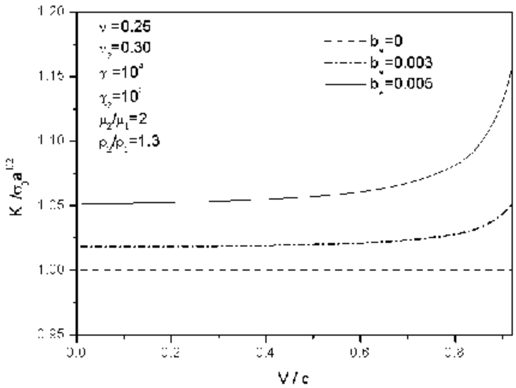

Fig. 4. Stress intensity factor $K_I / \sqrt{a}\sigma_0$ versus V/c for different values of b_e.

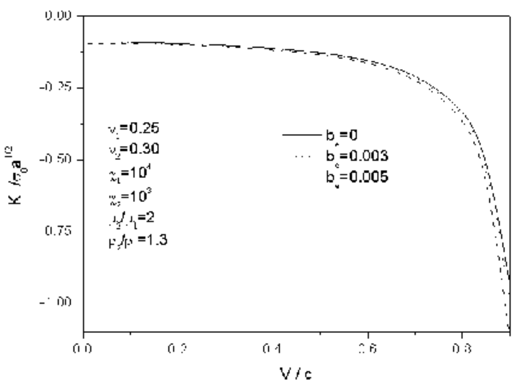

Fig. 5. Stress intensity factor $K_{II} / \sqrt{a}\sigma_0$ versus V/c for different values of b_e.

The same kind of results for $\chi_1 = 10^4$ and $\chi_2 = 10^3$ are shown in Figs. 4 and 5. It can be seen in the figures that the variation trends of $K_I / \sqrt{a}\sigma_0$ and $K_{II} / \sqrt{a}\sigma_0$ versus V/c are similar to those in Figs. 2 and 3. The magnitude of the dynamic stress intensity factors appear to depend on the smaller susceptibility. Of course, on increasing the magnetic induction to a larger value than those in Figs. 2 and 3 will increase the magnitude of the stress intensity factors.

Figs. 6 and 7 present the variations of the normalized stress intensity factors $K_I / \sqrt{a}\sigma_0$ and $K_{II} / \sqrt{a}\sigma_0$ at $V/c = 0.8$ and different susceptibility combinations as the magnetic induction changes. The results indicate that at given susceptibility combination, $K_I / \sqrt{a}\sigma_0$ increases while

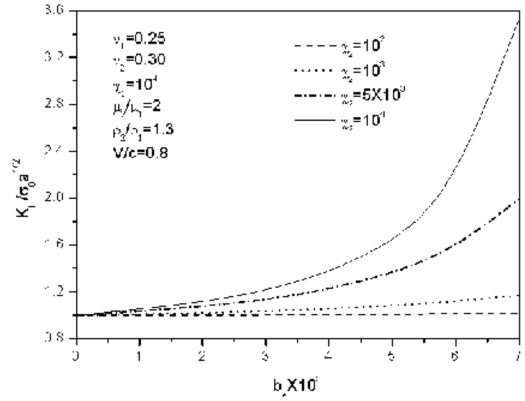

Fig. 6. Stress intensity factor $K_I / \sqrt{a}\sigma_0$ versus b_e for different susceptibility combinations.

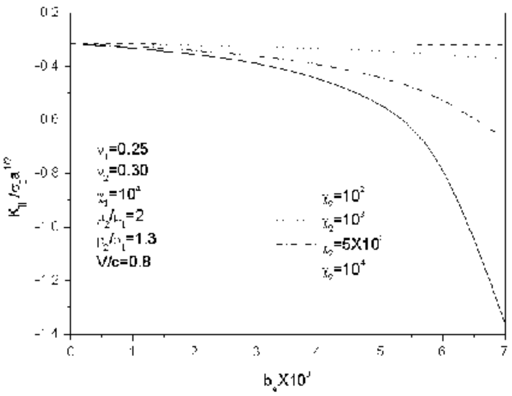

Fig. 7. Stress intensity factor $K_{II} / \sqrt{a}\sigma_0$ versus b_e for different susceptibility combinations.

$K_{II} / \sqrt{a}\sigma_0$ decreases slowly at lower magnetic induction and goes to change quickly at relatively stronger magnetic field. Changing the combination of both solids with different magnetic properties will affect the stress intensity factors obviously.

6. Conclusions

Closed-form solutions for the linear magnetoelastic problem of a moving interfacial crack between two bonded dissimilar soft ferromagnetic elastic materials under uniform magnetostatic field and uniform mechanical loading are presented by using the Yoffe's method and the integral transform technique. Results show that the effects of the velocity of the moving crack on the normalized dynamic stress intensity factors have the same tendencies as those in the corresponding purely elastic case. The concentration of stresses near the moving interfacial crack strongly depends on the external magnetic field, the magnetic susceptibilities of the bonded solids and their combinations.

References

Asanyan, D. D., Aslanyan, A. A. and Bagdasaryan, G. E., 1988, "Concentrantions of Elastic Stresses and Induced Magnetic Field Caused by an External Magnetic Field Next to a Crack," *Soviet Journal of Contemporary Engineering and Mechanics*, (I.A.N.A. SSR Mekhanika), Vol. 41, pp.10~19.

Bagdasarian, G. Y. and Hasanian D. J., 2000, "Magnetoelastic Interaction Between a Soft Ferromagnetic Elastic Half-plane with a Crack and a Constant magnetic field," *International Journal of Solids and Structures*, Vol. 37, NO. 38, pp. 5371~ 5383.

Erdogan, F. and Gupta, G. D., 1971, "Layered Composites with an Interface Flaw," *International Journal of Solids and Structures,* Vol. 7, No. 8, pp. 1089~1107.

Erdogan, F., 1984, "Mixed Boundary Value Problems in Mechanics," In: Nemat Nasser S. (Ed.), *Mechanics Today*, Vol. 4, Pergamon Press. Oxford.

Gradshteyn, I. S. and Ryzhik, I. M., 1980, *Table of Integrals, Series, and Products,* Academic Press, New York.

Karpenko, L. N., 1966, "Approximate Solution of a Singular Integral Equation by Means of Jacobi Polynomials," *Journal of Applied Mathematics and Mechanics,* Vol. 30, No. 3, pp. 668~675.

Liang, W., Shen, Y. P. and Zhao, M., 2000, "Magnetoelastic Formulation of Soft Ferromagnetic Elastic Problems with Collinear Cracks: Energy Density Fracture Criterion," *Theoretical and Applied Fracture Mechanics*, Vol. 34, No. 1, pp. 49~60.

Lin, C. B. and Lin, H. M., 2002, "The Magnetoelastic Problem of Cracks in Bonded Dissimilar Materials," *International Journal of Solids and Structures*, Vol. 39, No. 10, pp. 2807~2826.

Lin, C. B. and Yeh, C. S., 2002, "The Magne-toelastic Problem of a Crack in a Soft Ferromagnetic solid," *International Journal of Solids and Structures*, Vol. 39, No. 1, pp. 1~17.

Muskhelishvili, N. I., 1953, *Singular Integeal Equations*, Noordhoff, Groningen, The Netherlands.

Pao, Y. H. and Yeh, C. S., 1973, "A Linear Theory for Soft Ferromagnetic Elastic Solids," *International Journal of Engineering Science*, Vol. 11, No. 4, pp. 415~436.

Sabir, M. and Maugin, G. A., 1996, "On the Frac-ture of Paramagnets and Soft Ferromagnets," *Inter-national Journal of Non-Linear Mechanics*. Vol. 31, No. 4, pp. 425~440.

Shindo, Y., 1977, "The Linear Magnetoelastic Problem for a Soft Ferromagnetic Elastic Solid with a Finite Crack," *ASME Journal of Applied Mechanics*, Vol. 44, pp. 47~51.

Shindo, Y., 1982, "The Linear Magnetoelastic Pro-blem of Two Coplanar Griffith Cracks in a Soft Ferromagnetic Elastic Strip," *ASME Journal of Applied Mechanics,* Vol. 49, pp. 69~74.

Shindo, Y., 1983, "Dynamic Singular Stresses for a Riffith Crack in a Soft Ferromagnetic Elastic Solid Subjected to a Uniform Magnetic Field," *ASME Journal of Applied Mechanics,* Vol. 50, pp. 50~56.

Shindo, Y., 1984, "Diffraction of Waves and Sin-gular Stresses in a Soft Ferromagnetic Elastic Solid with Two Coplanar Griffith Cracks," *Journal of the Acoustical Society of America*, Vol. 75, pp. 50~57.

Yeh, C. S., 1989, "Magnetic Fields Generated by a Mechanical Singularity in a Magnetized Elastic Half Plane," *ASME Journal of Applied Mechanics,* Vol. 56, pp. 89~95.

Yoffe, E. H., 1951, "The Moving Griffith Crack," *Philosophical Magazine*. Vol. 42, pp. 739~750.

Zhao S. X. and Lee K. Y., 2004, "Eccentric Crack Problem in Soft Ferromagnetic Elastic Strip Under Uniform Magnetic Field," *Archive of Applied Me-chanics*. Vol. 73, No. 11-12, pp. 799~811.

Zhao S. X. and Lee K. Y., 2007, "Interfacial Crack Problem in Layered Soft Ferromagnetic Materials in Uniform Magnetic Field," *Mechanics Research Com-munications*. Vol. 34, pp. 19~30.

Appendices

$$k_1 = \frac{\nu_1}{1-2\nu_1} - \frac{1-\nu_1}{1-2\nu_1}\theta_1^{(1)}\gamma_1^{(1)}$$

$$k_2 = \frac{\nu_1}{1-2\nu_1} - \frac{1-\nu_1}{1-2\nu_1}\theta_2^{(1)}\gamma_2^{(1)}$$

$$k_3 = \frac{\chi_1 b_{e1}^2 \gamma_2^{(1)}}{\mu_{r1}}\left(1-2\nu_1\right), \quad \bar{k}_3 = k_3 + \frac{\chi_1^2 b_{e1}^2 \gamma_2^{(1)}}{2\mu_{r1}}$$

$$k_4 = \gamma_1^{(1)} + \theta_1^{(1)}$$

$$k_5 = \gamma_2^{(1)} + \theta_2^{(1)}$$

$$k_6 = \frac{\chi_1 b_{e1}^2}{\mu_{r1}}\left(4\nu_1 - 1\right) \tag{A.1}$$

$$k_7 = \frac{\nu_2}{1-2\nu_2} + \frac{1-\nu_2}{1-2\nu_2}\theta_1^{(2)}\gamma_1^{(2)}$$

$$k_8 = \frac{\nu_2}{1-2\nu_2} + \frac{1-\nu_2}{1-2\nu_2}\theta_2^{(2)}\gamma_2^{(2)}$$

$$k_9 = \frac{\chi_2 b_{e2}^2 \gamma_2^{(2)}}{\mu_{r2}}\left(2\nu_2 - 1\right),$$

$$\bar{k}_9 = \frac{\mu_2 \mu_{r1} \gamma_2^{(1)}}{\mu_1 \mu_{r2} \gamma_2^{(2)}}\left(k_9 - \frac{\chi_2^2 b_{e2}^2 \gamma_2^{(2)}}{2\mu_{r2}}\right)$$

$$k_{10} = \gamma_1^{(2)} - \theta_1^{(2)}$$

$$k_{11} = \gamma_2^{(2)} - \theta_2^{(2)}$$

$$k_{12} = \frac{\chi_2 b_{e2}^2}{\mu_{r2}}\left(1-4\nu_2\right), \quad \bar{k}_{12} = k_6 - \frac{\mu_2 \mu_{r1}\gamma_2^{(1)}}{\mu_1 \mu_{r2}\gamma_2^{(2)}} k_{12}$$

and

$$f_{11}^{(1)} = \frac{R_1}{R_1 R_3 - R_2 R_4},$$

$$f_{12}^{(1)} = \frac{1}{R_1 R_3 - R_2 R_4}\left(R_1 R_5 - R_4 R_6\right)$$

$$f_{21}^{(1)} = -\frac{R_2}{R_1 R_3 - R_2 R_4}$$

$$f_{22}^{(1)} = \frac{1}{R_1 R_3 - R_2 R_4}\left(R_3 R_6 - R_2 R_5\right)$$

$$f_{31}^{(1)} = -\frac{d_0}{d_1}\left(\theta_1^{(2)} f_{11}^{(2)} + \theta_2^{(2)} f_{21}^{(2)}\right)$$

$$f_{32}^{(1)} = -\frac{d_0}{d_1}\left(\theta_1^{(2)} f_{12}^{(2)} + \theta_2^{(2)} f_{22}^{(2)}\right) + \frac{1}{d_1}$$

$$f_{11}^{(2)} = \frac{1}{d_2 d_5 - d_3 d_4}\Big[\left(k_4 d_5 - k_1 d_3\right) f_{11}^{(1)} + \left(k_5 d_5 - k_2 d_3\right) f_{21}^{(1)}\Big] \tag{A.2}$$

$$f_{12}^{(2)} = \frac{1}{d_2 d_5 - d_3 d_4}\left[\left(k_4 d_5 - k_1 d_3\right) f_{12}^{(1)} + \left(k_5 d_5 - k_2 d_3\right) f_{22}^{(1)} + \frac{1}{d_1}\left(\bar{k}_{12} d_5 - (\bar{k}_3 + \bar{k}_9) d_3\right)\right]$$

$$f_{21}^{(2)} = \frac{1}{d_2 d_5 - d_3 d_4}\Big[\left(k_1 d_2 - k_4 d_4\right) f_{11}^{(1)} + \left(k_2 d_2 - k_1 d_4\right) f_{21}^{(1)}\Big]$$

$$f_{22}^{(2)} = \frac{1}{d_2 d_5 - d_3 d_4}\left[\left(k_1 d_2 - k_4 d_4\right) f_{12}^{(1)} + \left(k_2 d_2 - k_1 d_4\right) f_{22}^{(1)} + \frac{1}{d_1}\left((\bar{k}_3 + \bar{k}_9) d_2 - \bar{k}_{12} d_4\right)\right]$$

$$f_{31}^{(2)} = -\frac{\mu_{r1}\gamma_2^{(1)}}{\mu_{r2}\gamma_2^{(2)}} f_{31}^{(1)}, \quad f_{32}^{(2)} = -\frac{\mu_{r1}\gamma_2^{(1)}}{\mu_{r2}\gamma_2^{(2)}} f_{32}^{(1)}$$

where

$$R_1 = \theta_2^{(1)} - \frac{d_6}{d_2 d_5 - d_3 d_4}\Big[\left(k_5 d_5 - k_2 d_3\right)\theta_1^{(2)} + \left(k_2 d_2 - k_5 d_4\right)\theta_2^{(2)}\Big]$$

$$R_2 = \theta_1^{(1)} - \frac{d_6}{d_2 d_5 - d_3 d_4}\Big[\left(k_4 d_5 - k_1 d_3\right)\theta_1^{(2)} + \left(k_1 d_2 - k_4 d_4\right)\theta_2^{(2)}\Big]$$

$$R_3 = 1 - \frac{1}{d_2 d_5 - d_3 d_4}\Big[\left(d_5 - d_4\right)k_4 + \left(d_2 - d_3\right)k_1\Big]$$

$$R_4 = 1 - \frac{1}{d_2 d_5 - d_3 d_4}\Big[\left(d_5 - d_4\right)k_5 + \left(d_2 - d_3\right)k_2\Big] \tag{A.3}$$

$$R_5 = \frac{1}{d_1(d_2 d_5 - d_3 d_4)}\Big[\left(\bar{k}_{12} d_5 - (\bar{k}_3 + \bar{k}_9) d_3\right) + \left((\bar{k}_3 + \bar{k}_9) d_2 - \bar{k}_{12} d_4\right)\Big]$$

$$R_6 = \frac{d_6}{d_1(d_2 d_5 - d_3 d_4)}\Big[\left(\bar{k}_{12} d_5 - (\bar{k}_3 + \bar{k}_9) d_3\right)\theta_1^{(2)} + \left((\bar{k}_3 + \bar{k}_9) d_2 - \bar{k}_{12} d_4\right)\theta_2^{(2)}\Big] + \frac{1}{d_1}\left[\frac{2\chi_1 b_{e1}^2}{\mu_{r1}}\left(1 - 2\nu_1\right) + \frac{2\chi_2 \mu_{r1} b_{e2}^2 \gamma_2^{(1)}}{\mu_{r2}^2 \gamma_2^{(2)}}\left(1 - 2\nu_2\right)\right] - 1$$

and

$$d_0 = 1 - \frac{\chi_2 \mu_{r1}}{\chi_1 \mu_{r2}}$$

$$d_1 = -\frac{\mu_{r1}}{\chi_1}\left(1 + \frac{\mu_{r1}\gamma_2^{(1)}}{\mu_{r2}\gamma_2^{(2)}}\right) + \frac{2\chi_2 \mu_{r1} b_{e2}^2 \gamma_2^{(1)}}{\mu_{r2}^2 \gamma_2^{(2)}}\left(1 - 2\nu_2\right) d_0$$

$$d_2 = \frac{d_0}{d_1}\theta_1^{(2)}\bar{k}_{12} - \frac{\mu_2}{\mu_1} k_{10}$$

$$d_3 = \frac{d_0}{d_1}\theta_2^{(2)}\bar{k}_{12} - \frac{\mu_2}{\mu_1} k_{11} \tag{A.4}$$

$$d_4 = \frac{d_0}{d_1}\left(\bar{k}_3 + \bar{k}_9\right)\theta_1^{(2)} + \frac{\mu_2}{\mu_1} k_7$$

$$d_5 = \frac{d_0}{d_1}\left(\bar{k}_3 + \bar{k}_9\right)\theta_2^{(2)} + \frac{\mu_2}{\mu_1} k_8$$

$$d_6 = -\frac{d_0}{d_1}\left[\frac{2\chi_1 b_{e1}^2}{\mu_{r1}}\left(1 - 2\nu_1\right) + \frac{2\chi_2 \mu_{r1} b_{e2}^2 \gamma_2^{(1)}}{\mu_{r2}^2 \gamma_2^{(2)}}\left(1 - 2\nu_2\right)\right] + 1$$